Standards Compliance Testing of Optical Transmitters Using a Software-Based Equalizing Reference Receiver

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Abstract: Incorporation of EDC technology in optical networking presents new challenges in developing tests for transmitter standards compliance. This paper describes the 10GBASE-LRM TWDP test, recently adopted for 10Gbps Ethernet links over dispersive multimode fiber.

1. Introduction

As electronic dispersion compensation (EDC) becomes more prevalent in optical networks, new methods of testing the transmitter are required to ensure that a standards-compliant link will close. The 10GBASE-LRM standard [1], recently approved by the IEEE 802.3 Working Group, incorporated such a methodology in the form of the Transmitter Waveform and Dispersion Penalty (TWDP) test. Similar methodologies are under consideration for compliance testing in the T11.2 Fibre Channel and SFF8431 SFP+ committees. This paper discusses the motivation for this type of test, describes the TWDP test procedure, and provides supporting mathematical details of the underlying algorithm.

2. Motivation

The two-part challenge facing standards-setting bodies is to: 1) create specifications that can be used to separately qualify individual system components such that the components will interoperate satisfactorily when combined into a system, and 2) to do so in a manner that does not overly constrain the components, adding unnecessary cost to the system. For standardization of optical networks, that challenge is heightened by the introduction of receiver equalization in the form of electronic dispersion compensation (EDC). Receive equalization can compensate not only for fiber dispersion, but also for bandwidth limitations in the transmitter and receiver, offering potential cost savings in those components. To take advantage of this, compliance tests for those components must be defined such that impairments that cannot be equalized are screened out, while impairments that can be equalized are allowed. For the transmitter, that means defining a test other than a simple eye diagram mask. Conversely, some impairments that are not proscribed by an eye diagram mask may be problematic for a system designed to work over highly dispersive fiber with EDC employed at the receiver. For example, nonlinearities present in the transmitted waveform may be fine for a simple threshold receiver in the absence of fiber dispersion, but they may cause a failure of the link when combined with highly dispersive fiber and an equalizing receiver, since most common equalizer structures are designed for linear channels. For these reasons, the TWDP test was defined for 10GBASE-LRM. TWDP takes a holistic approach to transmitter qualification, combining effects of transmitter impairment with fiber propagation over reference fibers and equalization with a reference equalizing receiver.

10GBASE-LRM is not the first standard to employ this holistic approach of transmitter qualification. The IEEE 802.3ae committee defined a transmitter and dispersion penalty (TDP) test to check transmitter compliance that also took into account interactions of the transmitted waveform with a reference fiber into a reference receiver [2, Clause 52.9.10]. The significant difference between the TDP test and the TWDP test is that TDP employs actual hardware for the reference fiber and reference receiver (along with a reference transmitter, for comparison with the device under test). TWDP, on the other hand, implements the reference fiber and reference receiver in software, and compares performance with a theoretical SNR of an ideal transmitter, channel, and receiver. This eliminates the need for a costly instrument-grade reference transmitter, channel, and receiver, avoids the problem of calibrating those devices, and eliminates variability of those devices from one setup to another. On the other hand, TWDP requires an accurate digitization and capture of the transmitted waveform for use in software that emulates channel propagation and reception by the reference equalizing receiver. In practice, this digitization is usually accomplished with a high-bandwidth sampling oscilloscope.

3. TWDP Description

TWDP is a penalty defined as

$$\mathrm{TWDP} = \mathrm{SNR}_{REF} - \mathrm{SNR}_{TX} \quad \text{(dBo)} \tag{1}$$

where SNR_{REF} is a reference signal to noise ratio (SNR), expressed in optical dB (dBo), and SNR_{TX} is the equivalent SNR in dBo for the transmitter under test at the slicer input of a reference decision feedback equalizer (DFE) receiver for the measured waveform after propagation through a reference optical fiber channel. The transmitter under test is compliant with the standard if its computed TWDP is less than the upper limit specified by the 10GBASE-LRM standard.

3.1. Reference SNR

As used in 10GBASE-LRM, the reference SNR, SNR_{REF}, has a specific numeric value, and it is related to OMA_{RCV}, T, and N_0 through

$$\begin{aligned} \mathrm{SNR}_{\mathrm{REF}} &= \mathrm{OMA}_{\mathrm{RCV}} \sqrt{T/(2N_0)} \\ \mathrm{SNR}_{\mathrm{REF}}\ (\mathrm{dBo}) &= 10\log_{10}\left(\mathrm{OMA}_{\mathrm{RCV}} \sqrt{T/(2N_0)}\right) \end{aligned} \tag{2}$$

where OMA_{RCV} is the optical modulation amplitude (OMA) of the signal at the output of the reference channel, N_0 is the one-sided power spectral density of the additive white Gaussian noise (AWGN) assumed for the reference DFE receiver, and T is the bit duration, also called the unit interval.

Eqn. (2) gives the SNR that would be realized by an ideal matched filter receiver if the received waveform were an ideal rectangular NRZ waveform with amplitude OMA_{RCV} and the receiver had AWGN of spectral density N_0. This would be the SNR realized for a perfect NRZ transmitter, distortion-free fiber, and a theoretically optimal receiver.

The specific value of SNR_{REF} is set equal to some margin in optical dB above SNR_{RQD}, where SNR_{RQD} is the SNR required for the ideal matched filter receiver to achieve a desired bit error rate (BER) for an ideal NRZ received waveform. For the 10GBASE-LRM example, the desired BER is 10^{-12}, and SNR_{REF} is set 6.5 dBo above SNR_{RQD} to allow for 6.5 dB of optical power penalties related to dispersion. The BER for a such a matched filter receiver with an ideal NRZ input waveform is given by

$$\mathrm{BER} = Q(\mathrm{SNR}) \tag{3}$$

where $Q(\ )$ is the Gaussian error probability function

$$Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \tag{4}$$

(If SNR in Eqn. 3 were expressed as an electrical SNR, the argument of $Q(\ )$ would be $SNR^{1/2}$.)

With a target BER of 10^{-12}, Eqns. (3) and (4) result in $10\log_{10}(SNR_{RQD})$ equal to 8.47 dBo. Allowing for 6.5 dB of optical power penalties related to dispersion results in an SNR_{REF} given by

$$SNR_{REF} = 8.47 + 6.5 = 14.97 \text{ dBo} \tag{5}$$

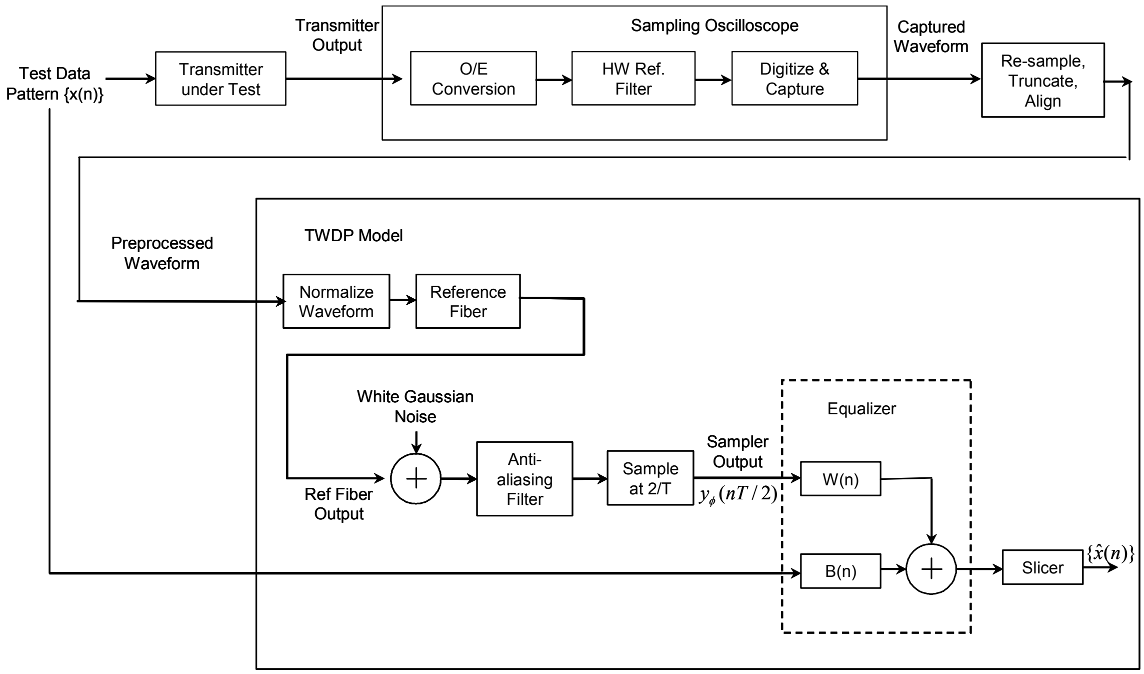
For a given OMA_{RCV}, SNR_{REF} determines a particular N_0 through Eqn. (2). Without loss of generality, the TWDP algorithm normalizes OMA_{RCV} to 1 and sets N_0 accordingly.

3.2. Waveform measurement and processing

Fig. 1 is used to describe both the waveform measurement and subsequent processing that calculates TWDP. The optical output of the transmitter under test is digitized and captured, for example, by a sampling oscilloscope with an appropriate head for O/E conversion. The data sequence driving the transmitter is a periodic PRBS9 or similar data pattern. The scope is set to capture the signal with at least seven samples per unit interval. The scope has a fourth-order Bessel-Thomson response with 3 dB electrical bandwidth of 7.5 GHz to filter the waveform (the "capture filter"). The scope is set to average out noise in the waveform.

The inputs to the algorithm that computes TWDP are the following:

- One complete cycle of the data sequence used to generate the transmitted waveform. The transmitted data sequence is denoted $\{x(n)\}$ and is periodic with period N (e.g., $N = 511$ for PRBS9)
- One complete cycle of the transmitted waveform, captured at 16 samples per unit interval. The captured waveform must be aligned with the data sequence (meaning that a rectangular pulse train based on the data sequence is aligned with the captured waveform within one unit interval). Depending on the scope



settings and capabilities, it may be necessary to preprocess the captured waveform to truncate it, resample it, and time align it with the data sequence before passing it to the algorithm.

The captured waveform is processed as follows:

1. The OMA and baseline (zero-level) of the captured waveform are estimated. The methods of estimating the baseline and OMA are described further below. The zero-level is subtracted from the waveform and the waveform is scaled such that the resulting OMA is 1. N_0 is set such that SNR_{REF} is 14.97 dBo, as described in the previous section. (Note: While the OMA at the *output* of the reference fiber is desired for Eqn. (2), the reference fiber is normalized to preserve OMA, so we may estimate the OMA at the transmitter output instead. The waveform is less distorted prior to fiber propagation, resulting in a more accurate estimate of the OMA.)

2. Three reference fiber channels are simulated, each corresponding to a defined stressor. The stressors were chosen as representative "bad" fibers that a 10GBASE-LRM link is designed to work over. The three stressors include a precursor impulse response, a post-cursor impulse response, and a split-symmetric impulse response. The waveform is passed though each of these simulated channels to compute a "Trial TWDP" for each channel. The TWDP reported is the maximum of the three Trial TWDP values. The remainder of this description describes the processing done for each simulated fiber.

3. In Figure 1, white Gaussian noise is shown added to the channel output and the sum is passed through an antialiasing filter. A fourth-order Butterworth filter of bandwidth 7.5 GHz is used as the antialiasing filter. The model in Figure 1 is useful for describing the TWDP calculation, but it does not depict the actual signal processing performed by the code. In the TWDP code, the signal (without added noise) is passed through the antialiasing filter to produce a deterministic output. The autocorrelation of the filtered noise is separately computed, and that autocorrelation is used to solve for the optimal equalizer coefficients and the equivalent SNR as described more fully below.

4. The antialiasing filter output is sampled at rate $2/T$ with phase ϕ. In the model, the input to the sampler consists of distorted signal plus noise. Denote the model output of the sampler at time $nT/2$ as $y_\phi(nT/2)$, which has a deterministic component and a random component given by

$$y_\phi(nT/2) = r(n) + \eta(n) \qquad (6)$$

The deterministic sequence $\{r(n)\}$ is computed by the TWDP code. $\{r(n)\}$ is periodic and is the sampled version of the filtered output of the reference fiber. $\{x(n)\}$ repeats after N bits, therefore $\{r(n)\}$ repeats after $2N$ samples (two samples per bit). The sequence $\{\eta(n)\}$ is a discrete-time noise sequence which would result from passing the white Gaussian noise through the anti-aliasing filter and sampling.

5. The sampled signal is processed by a fractionally-spaced minimum mean squared error decision feedback equalizer (MMSE-DFE) receiver with 14 feedforward taps at $T/2$ spacing ($W(n)$) and 5 feedback taps ($B(n)$). (The reference receiver does not imply a preferred implementation. Rather, it is simply a reference with a well-defined performance that provides a figure of merit for the transmitter when combined with the reference fiber channels.) A DFE would normally feed back decisions $\{\hat{x}(n)\}$. In this case, we assume that decisions are correct and replace the decided bits with the transmitted bits $\{x(n)\}$. The feedforward filter is augmented with a 15th tap that adjusts to optimize a constant offset. The feedforward and feedback tap coefficients are calculated using a least-squares approach that minimizes the mean-squared error at the slicer input for the given captured waveform, assuming the white Gaussian noise has a noise power spectral density of N_0 as determined in the previous section. The sampling phase and equalizer delay (described below) are also optimized. Mathematical details of this optimization are provided in a subsequent section.

6. Once the MSE is minimized by finding the optimal sampling phase, equalizer delay, and tap coefficients, the bit-error rate is calculated by the semi-analytic method, as follows:

    a. The Gaussian noise variance at the input to the slicer is calculated.

    b. For each bit in the data sequence, the equalized signal at the slicer input is calculated and the probability of error is determined based on the amplitude of the equalized signal, the variance of the noise at the slicer input, and the value of the slicer threshold.

    c. The probabilities of error are averaged over all slicer inputs (bits) in the sequence to compute a total probability of error $\mathrm{BER}_{\mathrm{TX}}$.

7. The equivalent SNR in optical dB is deduced from the $\mathrm{BER}_{\mathrm{TX}}$ as:

$$\mathrm{SNR}_{\mathrm{EQUIV}} = 10\log_{10}\left(Q^{-1}(\mathrm{BER}_{\mathrm{TX}})\right) \qquad (7)$$

8. The Trial TWDP for each simulated fiber is equal to the difference (in optical dB) between the reference SNR and the equivalent SNR, i.e.

$$\text{Trial TWDP} = \mathrm{SNR}_{\mathrm{REF}} - \mathrm{SNR}_{\mathrm{EQUIV}} \text{ (dBo)} \qquad (8)$$

    The TWDP reported is the maximum of the three Trial TWDP values.

4. Mathematical Details

This section provides supporting details for optimization of the decision feedback equalizer in the reference receiver, optimization of the constant offset coefficient, and estimation of OMA and baseline.

4.1. DFE Optimization

The DFE of the reference receiver is optimized in a manner that makes no assumption about the channel being linear. The algorithm finds the optimal tap settings that minimize mean-squared error when averaged over the entire data sequence, even when the waveform is nonlinearly related to the transmitted sequence.

The reference DFE shown in Fig. 1 consists of a feedforward filter $\{W(0),\ldots,W(N_f\text{-}1)\}$, a constant offset coefficient $W(N_f)$, and a feedback filter $\{B(1),\ldots,B(N_b)\}$, where $N_f = 14$ and $N_b = 5$. The feedback filter is symbol spaced and strictly causal, feeding back the five bits prior to the current bit $x(n)$. The input sequence on which the slicer makes decisions is denoted $\{z(n)\}$, where

$$\begin{aligned} z(n) &= \sum_{k=0}^{N_f-1} W(k) y_\phi(nT + DT - kT/2) + W(N_f) - \sum_{k=1}^{N_b} B(k)x(n-k) \\ &= \sum_{k=0}^{N_f-1} W(k)\big(r(2n+2D-k) + \eta(2n+2D-k)\big) + W(N_f) - \sum_{k=1}^{N_b} B(k)x(n-k) \end{aligned} \tag{9}$$

Note that the constant offset coefficient, $W(14)$, can be thought of as any other adjustable tap in the feedforward filter, except that it is fed by the constant 1 instead of a sample of $y(t)$. In (9), D is an integer such that the number of anticausal $T/2$-spaced taps in the feedforward filter is $2D$. DT is referred to as the equalizer delay. The equalizer delay can also be expressed as the number of feedforward anticausal taps, $2D$.

The least-squares solution for the feedforward and feedback filters minimizes the quantity

$$\text{MSE} = \text{E}\left[\sum_{n=0}^{N-1} \big(z(n) - x(n)\big)^2\right] \tag{10}$$

where the expectation operator, E, refers to the random sequence generated by the additive white Gaussian noise, filtered through the antialiasing filter and feedforward filter. (Here MSE is actually N times the mean-squared error when averaged over the input sequence $\{x(n)\}$.) The optimal filter coefficients are computed using the deterministic sequence $\{r(n)\}$ and the autocorrelation of the random sequence $\{\eta(n)\}$.

We first derive the MSE solution without offset optimization ($W(N_f)=0$). Assuming that the AWGN in Fig. 1 is mean zero, $\{\eta(n)\}$ is also mean zero. Denote the autocorrelation sequence for $\{\eta(n)\}$ by $\{c(k)\}$.

$$c(k) \equiv \text{E}\big[\eta(n)\eta(n-k)\big] = c(-k) \tag{11}$$

From Eqn. (9) with $W(N_f)=0$, the slicer input sequence $z(n)$ can be conveniently described in matrix notation for all values of n, $0 \le n \le N-1$, as

$$\mathbf{z} = \big(\mathbf{R} + \mathbf{N}\big)\mathbf{w} - \mathbf{X}\begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix} \tag{12}$$

where

$$\mathbf{z} \equiv \begin{bmatrix} z(0) \\ z(1) \\ \vdots \\ z(N-1) \end{bmatrix}; \qquad \mathbf{w} \equiv \begin{bmatrix} W(0) \\ W(1) \\ \vdots \\ W(N_f\text{ -}1) \end{bmatrix}; \qquad \mathbf{b} \equiv \begin{bmatrix} B(1) \\ B(2) \\ \vdots \\ B(N_b) \end{bmatrix} \tag{13}$$

$$\mathbf{R} \equiv \begin{bmatrix} r(2D) & r(2D-1) & \dots & r(2D-N_f+1) \\ r(2D+2) & r(2D+1) & \dots & r(2D-N_f+3) \\ \vdots & \vdots & & \vdots \\ r(2D+2(N-1)) & r(2D+2N-3) & \dots & r(2D+2(N-1)-N_f+1) \end{bmatrix} \tag{14}$$

$$\mathbf{N} \equiv \begin{bmatrix} \eta(2D) & \eta(2D-1) & \dots & \eta(2D-N_f+1) \\ \eta(2D+2) & \eta(2D+1) & \dots & \eta(2D-N_f+3) \\ \vdots & \vdots & & \vdots \\ \eta(2D+2(N-1)) & \eta(2D+2N-3) & \dots & \eta(2D+2(N-1)-N_f+1) \end{bmatrix} \tag{15}$$

$$\mathbf{X} \equiv \begin{bmatrix} x(0) & x(-1) & \ldots & x(-N_b) \\ x(1) & x(0) & \ldots & x(-N_b+1) \\ \vdots & \vdots & & \vdots \\ x(N) & x(N-1) & \ldots & x(N-N_b) \end{bmatrix} \tag{16}$$

For a given equalizer delay DT and sampling phase ϕ, minimizing MSE requires finding the vectors **w** and **b** that minimize the quantity

$$\text{MSE} = \text{E}\left[\left|\mathbf{z}-\mathbf{x}\right|^2\right] \tag{17}$$

where $\left|\bullet\right|^2$ indicates the squared magnitude of the vector argument. Here **x** is the vector of N bits in the periodic sequence $\{x(n)\}$

$$\mathbf{x} \equiv \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \tag{18}$$

From Eqn. (12) and the definitions of **z**, **x**, and **b**, it is seen that **z-x** can be expressed as

$$\mathbf{z}-\mathbf{x} = \left(\mathbf{R}+\mathbf{N}\right)\mathbf{w} - \mathbf{X}\tilde{\mathbf{b}} \tag{19}$$

where

$$\tilde{\mathbf{b}} \equiv \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix} \tag{20}$$

Using the fact that E[**N**] is the all-zeroes matrix, it follows that

$$\text{E}\left[\left|\mathbf{z}-\mathbf{x}\right|^2\right] = \mathbf{w}'\left(\mathbf{R}'\mathbf{R}+\mathbf{C}_\eta\right)\mathbf{w} - 2\mathbf{w}'\mathbf{R}'\mathbf{X}\tilde{\mathbf{b}} + \tilde{\mathbf{b}}'\mathbf{X}'\mathbf{X}\tilde{\mathbf{b}} \tag{21}$$

where $\mathbf{v}'$ indicates the transpose of the vector or matrix **v**, and

$$\mathbf{C}_\eta \equiv \text{E}\left[\mathbf{N}'\mathbf{N}\right] = N\begin{bmatrix} c(0) & c(1) & \cdots & c(N_f-1) \\ c(1) & c(0) & \cdots & c(N_f-2) \\ \vdots & \vdots & & \vdots \\ c(N_f-1) & c(N_f-2) & \cdots & c(0) \end{bmatrix} \tag{22}$$

(In the 10GBASE-LRM TWDP code, the matrix **C** equals $\mathbf{C}_\eta / N$.)

Minimizing the MSE in Eqn. (21) with respect to **w** for a given $\tilde{\mathbf{b}}$ gives

$$\mathbf{w}_{\text{opt}}\left(\tilde{\mathbf{b}}\right) = \left(\mathbf{R}'\mathbf{R}+\mathbf{C}_\eta\right)^{-1}\mathbf{R}'\mathbf{X}\tilde{\mathbf{b}} \tag{23}$$

Note that this result gives the optimal weight vector for a feedforward only equalizer by setting N_b=0. In that case $\tilde{\mathbf{b}} = 1$ and **X** = **x**.

Substituting the expression for $\mathbf{w}_{\text{opt}}\left(\tilde{\mathbf{b}}\right)$ for **w** in Eqn. (21) gives

$$\begin{aligned}
\mathrm{E}\left[\left|\mathbf{z}-\mathbf{x}\right|^2\right] &= \tilde{\mathbf{b}}'\,\tilde{\mathbf{P}}\,\tilde{\mathbf{b}} \\
&= \begin{bmatrix}1 & \mathbf{b}'\end{bmatrix}\begin{bmatrix}P_{00} & \mathbf{p}' \\ \mathbf{p} & \mathbf{P}\end{bmatrix}\begin{bmatrix}1 \\ \mathbf{b}\end{bmatrix} \\
&= P_{00} + 2\mathbf{p}'\mathbf{b} + \mathbf{b}'\,\mathbf{P}\,\mathbf{b}
\end{aligned} \tag{24}$$

where

$$\tilde{\mathbf{P}} \equiv \mathbf{X}'\,\mathbf{X} - \mathbf{X}'\,\mathbf{R}\left(\mathbf{R}'\,\mathbf{R} + \mathbf{C}_\eta\right)^{-1}\mathbf{R}'\,\mathbf{X} \tag{25}$$

Minimizing the resulting expression for MSE with respect to b gives

$$\mathbf{b}_{\mathrm{MMSE}} = -\mathbf{P}^{-1}\,\mathbf{p} \tag{26}$$

Collecting the results for the final answer, the minimizing feedback vector b is

$$\mathbf{b}_{\mathrm{MMSE}} = -\mathbf{P}^{-1}\,\mathbf{p} \tag{27}$$

where P is the square matrix in the lower right corner of $\tilde{\mathbf{P}}$ that excludes the first column and first row of $\tilde{\mathbf{P}}$, p is the first column of $\tilde{\mathbf{P}}$ excluding the top element P_{00}, and $\tilde{\mathbf{P}}$ is given by

$$\tilde{\mathbf{P}} \equiv \mathbf{X}'\,\mathbf{X} - \mathbf{X}'\,\mathbf{R}\left(\mathbf{R}'\,\mathbf{R} + \mathbf{C}_\eta\right)^{-1}\mathbf{R}'\,\mathbf{X} \tag{28}$$

The minimizing feedforward vector w is given by

$$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{R}'\,\mathbf{R} + \mathbf{C}_\eta\right)^{-1}\mathbf{R}'\,\mathbf{X}\begin{bmatrix}1 \\ \mathbf{b}_{\mathrm{MMSE}}\end{bmatrix} \tag{29}$$

The minimum MSE is given by

$$\begin{aligned}
\mathrm{MMSE} &= P_{00} + 2\mathbf{p}'\mathbf{b}_{\mathrm{MMSE}} + \mathbf{b}'_{\mathrm{MMSE}}\,\mathbf{P}\,\mathbf{b}_{\mathrm{MMSE}} \\
&= P_{00} - \mathbf{p}'\mathbf{P}^{-1}\mathbf{p}
\end{aligned} \tag{30}$$

The variance of the Gaussian noise at the input to the slicer, used to compute the BER semianalytically, is given by

$$\sigma^2 = \frac{1}{N}\mathbf{w}'_{\mathrm{MMSE}}\,\mathbf{C}_\eta\,\mathbf{w}_{\mathrm{MMSE}} \tag{31}$$

The description thus far has focused on optimization of the equalizer tap coefficients. With a finite length equalizer, it is also necessary to optimize the equalizer delay DT and the sampling phase ϕ. While the equalizer delay could be optimized in a brute force manner by trying different numbers of anticausal taps in the feedforward filter, the TWDP algorithm instead implements an efficient optimization, details of which can be found in [3]. The optimal sampling phase ϕ for the $T/2$ sampler is found by a brute-force search over the 16 sampling phases in the unit interval.

4.2. Offset Optimization

Including the extra feedforward tap $W(N_f)$ for automatic offset optimization requires a modification to the derivation above. The idea is to add an extra tap to the feedforward equalizer that is fed with the constant value 1 instead of a sample of the received signal. Optimizing the taps as before has the effect of adding the constant offset to the input of the slicer that minimizes mean squared error.

Augment the feedforward vector **w** with an extra tap to give the new feedforward vector $\tilde{\mathbf{w}}$

$$\tilde{\mathbf{w}} = \begin{bmatrix}\mathbf{w} \\ W\left(N_f\right)\end{bmatrix} \tag{32}$$

Augment the matrix of noiseless channel outputs R with an additional column of all ones, denoted as the column vector **1**, to give the new matrix

$$\tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{R} & \mathbf{1} \end{bmatrix} \tag{33}$$

Augment the matrix of noise samples **N** with an additional column of all zeros, denoted as the column vector **0**, to give the new matrix

$$\tilde{\mathbf{N}} = \begin{bmatrix} \mathbf{N} & \mathbf{0} \end{bmatrix} \tag{34}$$

Then

$$\tilde{\mathbf{C}}_\eta \equiv \mathrm{E}\left[\tilde{\mathbf{N}}'\tilde{\mathbf{N}}\right] = \begin{bmatrix} \mathbf{C}_\eta & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} \tag{35}$$

The solution for the case with automated offset optimization is obtained by replacing **w** with $\tilde{\mathbf{w}}$, **R** with $\tilde{\mathbf{R}}$, and $\mathbf{C}_\eta$ with $\tilde{\mathbf{C}}_\eta$ in Eqns. (27) through (31). Note that the relationships

$$\left(\tilde{\mathbf{R}}'\tilde{\mathbf{R}} + \tilde{\mathbf{C}}_\eta\right)^{-1} = \begin{bmatrix} \mathbf{R}'\mathbf{R} + \mathbf{C}_\eta & \mathbf{R}'\mathbf{1} \\ \mathbf{1}'\mathbf{R} & N \end{bmatrix}^{-1}; \qquad \tilde{\mathbf{w}}'\tilde{\mathbf{C}}_\eta\,\tilde{\mathbf{w}} = \mathbf{w}'\mathbf{C}_\eta\,\mathbf{w} \tag{36}$$

are used in the 10GBASE-LRM code that computes TWDP.

4.3. OMA and baseline estimation

The first step in the TWDP algorithm removes the baseline (zero-level) from the waveform and normalizes the OMA to 1. While IEEE 802.3aq (the committee that developed the 10GBASE-LRM standard) considered having OMA and baseline as inputs to the TWDP algorithm, it was decided that the algorithm itself should estimate these parameters to ensure a standard estimation method avoiding problems with TWDP measurement repeatability. The method of estimation relies on finding a linear approximation to the transmitted waveform and using that approximation to construct a low-frequency square wave from which the OMA and baseline can be extracted. The notation in this section differs somewhat from that in previous sections.

Let the transmitter optical output (filtered by the Bessel-Thomson capture filter) be denoted by $y(t)$. A simple linear (strictly, affine) model of $y(t)$ is given by

$$y_{\mathrm{lin}}(t) = b + \sum_{n=-\infty}^{\infty} x(n)q(t - nT) \tag{37}$$

where b is a constant baseline value, $x_n \in \{0,1\}$, $q(t)$ is the pulse shape output by the laser, and T is the bit period. We wish to find the b and $q(t)$ in Eqn. (37) such that $y_{\mathrm{lin}}(t)$ approximates $y(t)$. This problem is solved by finding the least-squares estimate that minimizes the mean squared error between $y(t)$ and $y_{\mathrm{lin}}(t)$. This solution can be approximated by transitioning to the discrete time domain and using linear algebra techniques.

Let $t = mT + k\Delta$, where $0 \le k \le K-1$ and $\Delta = T/K$. Eqn. (37) can be generalized slightly by allowing b to be a periodic function of t with period T. From Eqn. (37),

$$\begin{aligned} y_{\mathrm{lin}}(mT + k\Delta) &= b(k\Delta) + \sum_{n=-\infty}^{\infty} x(n)q\left((m-n)T + k\Delta\right) \\ &= b(k\Delta) + \sum_{n=-\infty}^{\infty} x(m-n)q\left(nT + k\Delta\right) \qquad k \in \{0,\ldots,K-1\} \end{aligned} \tag{38}$$

Assume that the pulse response is of finite duration such that $q(t) = 0$ for t outside the interval $\left[-AT, (M+1)T\right)$. Here A is the anticipation and M is the memory of the transmitter output in bit periods. Then, in the last line of Eqn. (38), the summation index n can be restricted to the range $-A \le n \le M$.

Let $\mathbf{y}'_m \equiv \left[y(mT) \quad y(mT+\Delta) \quad \cdots \quad y(mT+(K-1)\Delta) \right]$, with similar definitions for $\mathbf{q}_m$ and $\mathbf{y}_{\text{lin},m}$. Let $\mathbf{b}' \equiv \left[b(0) \ldots b((K-1)\Delta) \right]$ and $\mathbf{x}'_m \equiv \left[x(m+A) \ldots x(m) \ldots x(m-M) \right]$. Eqn. (38) can be compactly expressed for all K sampling phases by

$$\begin{aligned} \mathbf{y}_{\text{lin},m} &= \mathbf{b} + \mathbf{q}_{-A} x(m+A) + \ldots + \mathbf{q}_M x(m-M) \\ &= \mathbf{b} + \mathbf{Q}\mathbf{x}_m \end{aligned} \tag{39}$$

where $\mathbf{Q}$ is a K by $A+M+1$ matrix with columns $\mathbf{q}_{-A}$ through $\mathbf{q}_M$. Note that each column of Q consists of K successive samples of the continuous-time pulse response $q(t)$.

Now find the b and Q that minimize E[|$\boldsymbol{\varepsilon}$|^2], where

$$\boldsymbol{\varepsilon} \equiv \mathbf{y} - \mathbf{y}_{\text{lin},m} \tag{40}$$

|$\boldsymbol{\varepsilon}$|^2 is the magnitude squared of the vector $\boldsymbol{\varepsilon}$, and $\mathrm{E}[\bullet]$ indicates expected value. Treat **x, y** and $\boldsymbol{\varepsilon}$ as random vectors and drop the m subscript. The orthogonality principle for least squares estimation implies that

$$\mathrm{E}\left[\boldsymbol{\varepsilon}\mathbf{x}'\right] = \mathrm{E}\left[\left(\mathbf{y} - \left(\mathbf{b} + \mathbf{Q}^*\mathbf{x}\right)\right)\mathbf{x}'\right] = \mathbf{0} \tag{41}$$

when $\mathbf{Q}^*$ is the optimal **Q** for a given **b**. The least squares solution gives

$$\mathbf{Q}^* = \boldsymbol{\rho}\mathbf{R}_x^{-1} - \mathbf{b}\,\mathrm{E}\left[\mathbf{x}'\right]\mathbf{R}_x^{-1} \tag{42}$$

where

$$\mathbf{R}_x \equiv \mathrm{E}\left[\mathbf{x}\mathbf{x}'\right] \tag{43}$$

and

$$\boldsymbol{\rho} \equiv \mathrm{E}\left[\mathbf{y}\mathbf{x}'\right] \tag{44}$$

Using $\mathbf{Q}^*$ from Eqn. (42) in an expression for E[|$\boldsymbol{\varepsilon}^2$|] and minimizing with respect to **b** gives the optimal **b*** as

$$\mathbf{b}^* = \frac{\overline{\mathbf{y}} - \boldsymbol{\rho}\mathbf{R}_x^{-1}\overline{\mathbf{x}}}{1 - \overline{\mathbf{x}}'\mathbf{R}_x^{-1}\overline{\mathbf{x}}} \tag{45}$$

where an overbar indicates expected value. Substituting **b*** for **b** in Eqn. (42) gives the optimal **Q** for a periodic baseline that is allowed to vary with sampling phase. (The optimal *constant* bias b^* is the average of the components of **b***; details are not presented here.)

We now translate the results above, which are expressed in terms of expectation values, into calculations using the captured deterministic waveform and input sequence. Let $\mathbf{y}_m$ be the captured samples as defined above, where each vector $\mathbf{y}_m$ consists of K samples of a single bit period of the sampled waveform for a repetitive data test pattern of length N. Let $\mathbf{x}_m$ also be defined as above, where $\mathbf{x}_m$ is a vector consisting of A data bits preceding the "current" bit, the current bit $x(m)$, and M data bits after the current bit. Form the matrices **Y** and **X** with columns $\mathbf{y}_m$ and $\mathbf{x}_m$, respectively, where m varies from 0 to N-1. (Use the fact that $x(n)$ is periodic with period N when n is outside the range [0,...,N-1].) With expectations taken over the N data vectors $\mathbf{x}_m$, each assumed equally likely, the expected values required for the least squares fit above are given by

$$\mathbf{R}_x = \frac{1}{N}\mathbf{X}\mathbf{X}' \tag{46}$$

$$\boldsymbol{\rho} = \frac{1}{N}\mathbf{Y}\mathbf{X}' \tag{47}$$

$$\overline{\mathbf{x}} = \mathbf{X1}/N \tag{48}$$

$$\overline{\mathbf{y}} = \mathbf{Y1}/N \tag{49}$$

where **1** is an all-ones column vector of length *N*.

Eqn. (39) can be written compactly for all *N* bit periods as

$$\mathbf{Y}_{\text{lin}} = \begin{bmatrix}\mathbf{Q} & \mathbf{b}\end{bmatrix}\begin{bmatrix}\mathbf{X} \\ \mathbf{1}'\end{bmatrix} \tag{50}$$

The optimal Q* and b* can be found using a pseudo-inverse and the captured waveform matrix **Y**:

$$\begin{bmatrix}\mathbf{Q}^* & \mathbf{b}^*\end{bmatrix} = \mathbf{Y}\begin{bmatrix}\mathbf{X}' & \mathbf{1}\end{bmatrix}\left[\begin{bmatrix}\mathbf{X} \\ \mathbf{1}'\end{bmatrix}\begin{bmatrix}\mathbf{X}' & \mathbf{1}\end{bmatrix}\right]^{-1} \tag{51}$$

With some manipulation and making use of the relations in Eqns. (46) through (49), it can be shown that Eqn. (51) gives the same result as Eqns. (42) and (45) for the periodic bias case.

With the pulse response $q(t)$ and periodic bias (or baseline) thus estimated, the TWDP algorithm uses a linear approximation to synthesize an optical output for a periodic square wave of eight consecutive ones followed by eight consecutive zeros. The amplitude of the optical signal at the center of the “on” or “high” portion of the square wave is then measured by averaging over the center 20% of the “on” portion, and the amplitude of the optical signal at the center of the “off” or “low” portion of the square wave is similarly measured. The OMA estimate is then the difference between the “on” amplitude and the “off” amplitude, and the baseline estimate is the measurement of the “off” amplitude. This estimation technique attempts to approximate the measurement method specified in [2, Clause 52.9.1.2], which specifies a square wave OMA test pattern “consisting of four to eleven consecutive ones followed by an equal run of zeros”.

5. Summary

We have described a new transmitter compliance test methodology that was adopted as part of the 10GBASE-LRM standard by the IEEE 802.3 Working Group. The need for this new test methodology, which incorporates a software simulation of channel propagation and a reference equalizing receiver was discussed. Detailed mathematical derivations were presented to support the code that is written in the standard.

6. References

[1] IEEE Draft P802.3aq/D4.0, ..., Amendment: Physical Layer and Management Parameters for 10 Gb/s Operation, Type 10GBASE-LRM

[2] IEEE Std 802.3-2005 IEEE Standard for Information technology...Part 3: Carrier sense multiple access with collision detection (CSMA/CD) access method and physical layer specifications

[3] P. A. Voois, I. Lee, and J. M. Cioffi, “The effect of decision delay in finite-length decision feedback equalization,” IEEE Transactions on Information Theory, vol. 53, pp. 618-621, Mar. 1996.